Geometric and Physical Constraints Synergistically Enhance Neural PDE Surrogates

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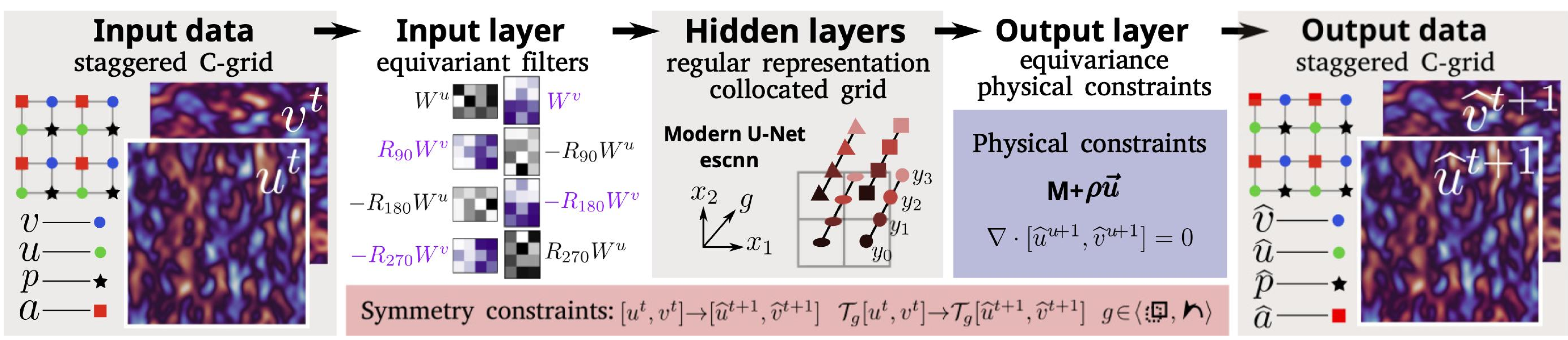
Introduction & Motivation

- **Challenges for ML-based PDE prediction:** long-term rollout accuracy, stability, generalization.
- **Previous ML Models:** can be improved by both **physical and geometric constraints.**
- **Staggered C-grids:** often used in weather, climate and fluid dynamics, but unsupported by standard equivariant convolution layers.
- **Our aim:** **impose these constraints on C-grids, and evaluate whether their benefits can be usefully combined.**

Symmetry– and Physics–Constrained Neural Surrogates

Network Architecture



The input- and output data are on C-grids. Starting from a strong modern U-net base architecture (Gupta et al., 2023), we imposed physical and symmetry constraints, individually and in combination.

Experiments

Shallow Water Equations (SWEs)

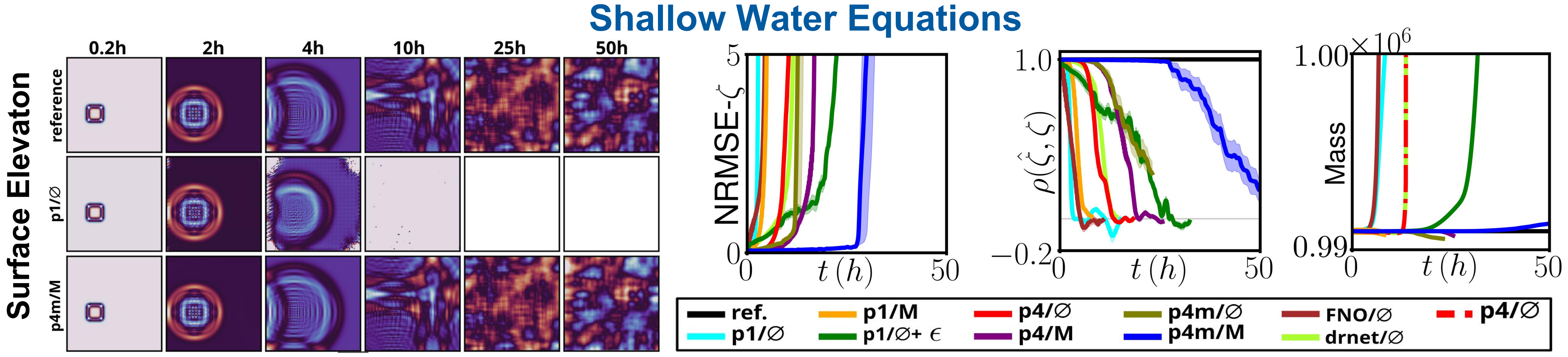
Conservation laws	Symmetries		
None ∅	p1/∅	p4/∅	p4m/∅
Mass **M**	p1/M	p4/M	p4m/M

Incompressible Navier–Stokes (INS) & Decaying Turbulence

Conservation laws	Symmetries		
None ∅	p1/∅	p4/∅	p4m/∅
Momentum $\rho\vec{u}$	p1/$\rho\vec{u}$	p4/$\rho\vec{u}$	p4m/$\rho\vec{u}$
Mass/momentum M+$\rho\vec{u}$	p1/M+$\rho\vec{u}$	p4/M+$\rho\vec{u}$	p4m/M+$\rho\vec{u}$

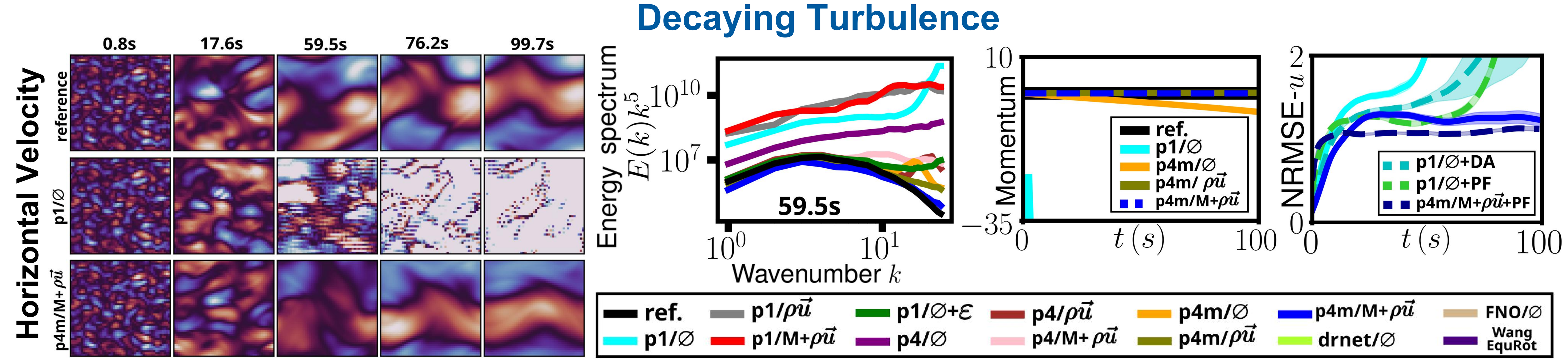
Double–Constrained Models Outperform Other Networks on SWEs & INS

Shallow Water Equations



- p4m/M (symmetry+physics constraints) outperforms other networks.
- Normalized RMSE and correlation are shown for 50h rollouts, with standard error of the mean over 20 ICs.

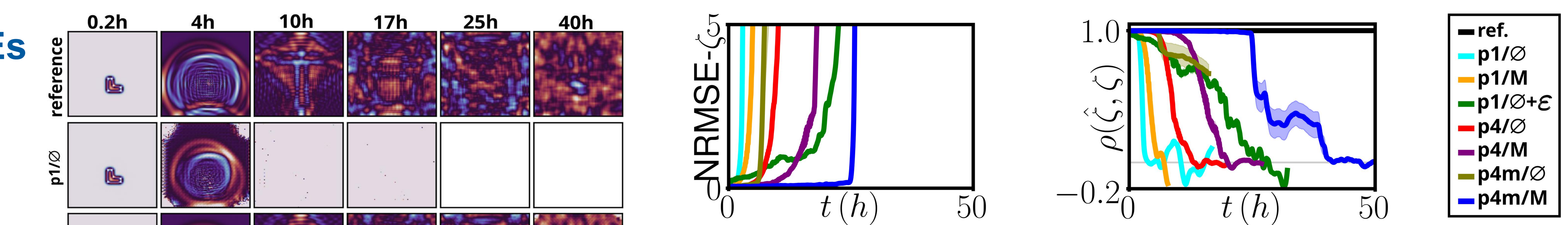
Decaying Turbulence



- p4m/M+ρu⃗ (symmetry+physics constraints) outperforms other networks with similar parameter counts on INS.
- Accuracy of energy spectrum and momentum with standard error of the mean over 30 ICs.

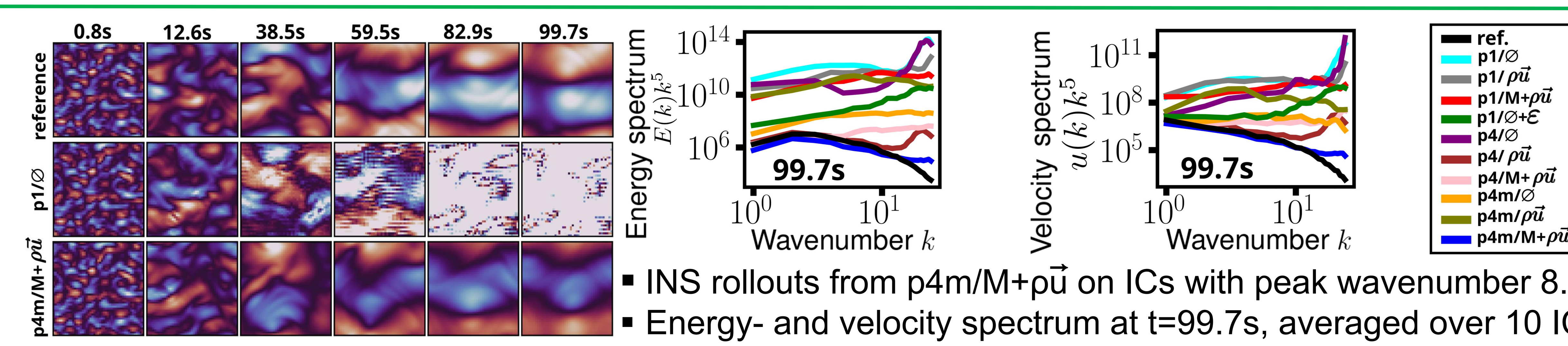
Generalization Beyond Training Data & Real Ocean Currents

SWEs



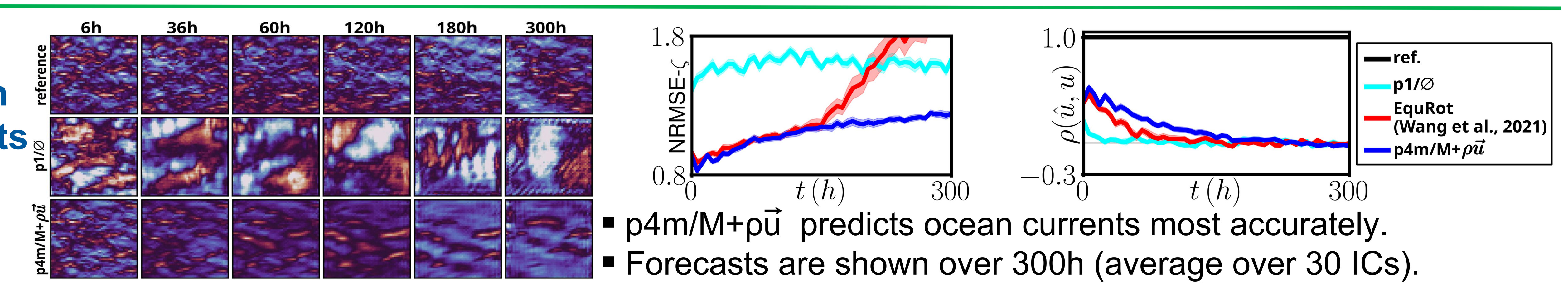
- SWEs rollouts from p4m/M for L-shaped initial disturbances.
- Accuracy of each network is computed over 6 ICs.

INS



- INS rollouts from p4m/M+ρu⃗ on ICs with peak wavenumber 8.
- Energy- and velocity spectrum at t=99.7s, averaged over 10 ICs.

Real ocean currents



- p4m/M+ρu⃗ predicts ocean currents most accurately.
- Forecasts are shown over 300h (average over 30 ICs).

Conclusion

- **We implemented the first physics+symmetry-constrained model on C-grids.**
- **Symmetries are more effective than physical constraints, but combining both is best.**
- **Our model improves generalisation to new ICs, and accurately predicts real ocean currents.**